Eqⁿ of Motion for Angular Momentum :-

The angular momentum 'L'

does not commute with Hamiltonian in Relativistic eq^n (Dirac eq^n).

$\vec{L} = \vec{r} \times \vec{p}$ is not conserved even for a free particle.

$$\frac{d\vec{L}}{dt} = \frac{i}{\hbar}[H, \vec{L}] = \frac{i}{\hbar} c [\vec{\alpha}\cdot\vec{p}, \vec{r}\times\vec{p}]$$

$$= c[\vec{\alpha}\cdot\nabla\vec{r}\times\vec{p}] = \underbrace{c\vec{\alpha}\times\vec{p}}$$

This is not going to be zero, thus angular momentum is not conserved.
Classically it is possible but quantum mechanically it is not possible i.e; angular momentum will be conserved always.

$$c\vec{\alpha} \times \vec{p} = \underbrace{\vec{v}\times\vec{v}}_{\text{must be zero}} = 0 \text{ but not zero}$$

The reason of this is that the velocity operator $(c\vec{\alpha})$ is different than the momentum operator $\vec{p}$.

Hence, angular momentum is not conserved.

Now, what do we do to make the angular momentum to be zero. Since, the basic property of angular momentum to be zero, should not be destroyed.

For this Dirac defined a new operator which is different than classical $\vec{L}$ and extra contribution is added to $\vec{L}$ to that angular momentum in define $\vec{J}$ and $\vec{J}$ is conserved.

$\vec{J} = \vec{L} + \vec{S}$ is conserved

This extra contribution is called the spin of the particle. This is intrinsic angular momentum which is not due to the motion of the particle in space and time but it is internal

property of every particle that spin angular momentum is to be added with $(\vec{L})$ and this $(\vec{L}+\vec{S})$ is known as total angular momentum $\vec{J}$.
Now, $\vec{J}$ is conserved.

In this position,

$$\frac{d\vec{J}}{dt} = \frac{d}{dt}(\vec{L}+\vec{S}) = 0$$

eigen value of $\vec{S} = \pm \frac{\hbar}{2}$

Therefore, the particle obeying Dirac are known as Spin half particle.

(What is the resultant angular momentum according Dirac's relativistic equation)